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HEAT TRANSFER AND FLUID FLOW PAST A CIRCULAR CYLINDER BETWEEN PARALLEL WALLS

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ABSTRACT
This paper presents a numerical study on heat transfer and fluid flow for unsteady, incompressible, Newtonian fluid past a circular cylinder in between confined walls, based on Thompson, Thames, Mastin (TTM) method of automatic boundary fitted co-ordinate generation system. Finite difference implicit method, based on vorticity-stream function formulation with uniform inlet flow conditions, is employed for unsteady state computations for Reynolds numbers in the range 50-150. Time-dependent periodic behavior of stream lines for Reynolds number from 50 up to 150 shows periodicity of flow field. The time-evolution of Nusselt number, averaged over the surface of the cylinder, for different Reynolds numbers are shown. The frequency of vortex shedding and the amplitude of the fluctuating average Nusselt number increases with Reynolds number. The proximity of the cylinder to a wall has considerable influence on the flow as well as heat transfer.

KEYWORDS: Circular Cylinder, Finite Difference Method, Parallel Walls, Heat Transfer, Nusselt Number.

I. INTRODUCTION
The main object of this study to investigate heat transfer and fluid flow past a circular cylinder between parallel walls. The uniform flow past a circular cylinder in confined walls has been investigated by many researchers both experimentally and numerically due to the practical importance of the flow. This configuration is found in many applications like cross flow heat exchangers. The flow in this type is characterized by the cylinder diameter (D), the free stream velocity (U) and the Reynolds number (Re). For flow past a circular cylinder, Taneda [1] investigated that the separation of boundary layer on the cylinder surface begins at Reynolds number equal to 5. Between Reynolds numbers 10 to 40, pair of steady symmetric vortices develops behind the cylinder and the length of re-circulation zone grows linearly as Reynolds number increases Beaudan [2]. He established that vortex shedding occurs for Reynolds number above 49 and the vortex shedding flow remains laminar for Reynolds number up to around 150. Williamson [3] found that transition to three dimensional flow starts at Reynolds number of around 180-194 depending on experimental condition and ends at Reynolds number equal to about 260 at which fine scale three dimensional eddies appear. The vortex shedding is regular and the Strouhal number, which represents vortex shedding frequency, remains unchanged. Singha [4] studied heat transfer and fluid flow for a cylinder at unbounded domaim. He found that frequency of vortex shedding and the amplitude of the fluctuating average Nusselt number increases with Reynolds number. However, when a circular cylinder is placed near a plane wall, the separation and wake development depend on the Reynolds number, the gap ratio and the characteristics of the boundary layer of the wall Bearman [5]. Lie et al [6] investigated vortex shedding near a plane wall for different gap ratios and for different Reynolds numbers ranging from 80 up to 1000. They observed vortex-shedding phenomenon using various methods. Singha et al [7] studied vortex shedding suppression and heat transfer for flow past a single cylinder near a plane wall. They found a critical gap ratio on which vortex shedding suppressed. In study, Zovatto and Pedrizzetti [8] have also studied the onset of vortex formation and the loss of stability of flow for a cylinder confined in between two walls for blockage ratio of 0.2. The study is based on the parabolic velocity profile of the liquid at the inlet to the channel.

Sahin & Owens [9] used a finite volume based method on a velocity only formulation to solve the flow field around a circular cylinder confined in a channel. They concluded that asymmetric vortex shedding is delayed with increasing blockage ratio. In their experiment, Wen & Lin [10] investigated the relationship of two

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dimensional vortex shedding frequency with the Reynolds number ranging from 45 up to 560. They used both horizontal and vertical soap film tunnels to set up the 2-D experiments. They have found good agreement of St-Re curve with other works. Vortex shedding past a circular cylinder under the influence of buoyancy has been studied by Patnaik et. Al. [11]. They have used finite element method with modified velocity correction procedure. They presented the influence of buoyancy on Nusselt number, wake structure etc. They found that at low Reynolds number about Re = 20-40 buoyancy opposing the flow could trigger vortex shedding.

In the present work, the interaction between a laminar stream and a circular cylinder placed between parallel walls is studied. In this study, the two dimension Navier-Stokes equations are solved in the finite difference implicit method with vorticity - stream function formulation such that continuity equation is satisfied exactly and pressure term is eliminated. The energy equation is also solved to find the heat transfer aspects. The value of Reynolds number of this study is so selected to avoid any instability of flow to three dimensional disturbances.

II. MATHEMATICAL FORMULATION

The flow domain of interest and boundary conditions are shown in Figure 1. A rectangular flow field with top and bottom as solid plane wall, contains a circular cylinder of diameter D, whose position from bottom no-slip wall is defined by the gap G. The cylinder is located at 5D from the inflow boundary. The top no-slip boundary is set at distance H=5D fixed from the bottom wall. The outflow boundary is located at 27D from the cylinder centre.

Consider an incompressible fluid, with density ρ and kinematics viscosity ν, flowing with steady velocity U in the flow domain. The problem is made dimensionless by taking D as unit length, D/U as unit time and ρD^3, as unit mass. The flow is governed by two dimensionless parameters, the Reynolds number Re, the blockage ratio d = D/H which is fixed in this case and the gap ratio G/D. The gap is a positive number which, for symmetry reasons, takes its maximum value = (1 − d)=2d when the cylinder is placed in the centre of between the walls. The Reynolds number and the gap ratio G/D. The gap ratio is a positive number which takes its minimum value zero as the cylinder touches the bottom wall.

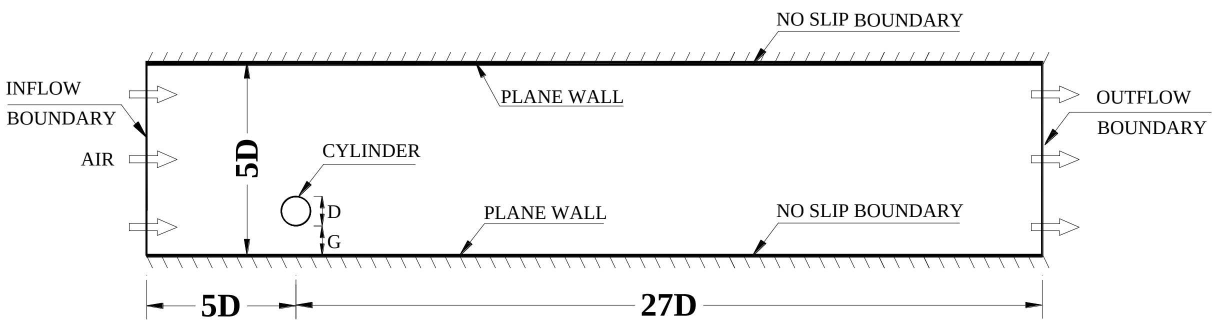

Fig.1 Flow domain and boundary conditions of the flow

In this study a Cartesian systems of co-ordinates (x, y) with x-axis along the bottom wall is considered. The governing equations are the Navier-stokes equations which are written in the vorticity-stream function formulation:

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \quad (1)$$

Vorticity and stream function are related by the poission's equation:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\omega \quad (2)$$

Where ω is the vorticity, ψ is the stream function and u, v denotes the components of velocity in x and y direction, which can be calculated from the stream function:

$$\frac{\partial \Psi}{\partial y} = u \qquad \frac{\partial \Psi}{\partial x} = -v \qquad (3)$$

The non dimensional energy equation in Cartesian co-ordinate system can be written as:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\text{Re}\,\text{Pr}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

III. NUMERICAL METHOD

In order to solve the governing equation in a curvilinear co-ordinate system, the following co-ordinate transformation is used:

$$\xi = \xi(x, y), \quad \eta = \eta(x, y) \qquad (5)$$

where (ξ, η) is the co-ordinate system in the computational plane. Using above transformation, the derivatives in the physical and computational plane are related as follows:

The Laplace equations used as the generation system are:

$$\xi_{xx} + \xi_{yy} = P(\xi, \eta) \qquad (5)$$

$$\eta_{xx} + \eta_{yy} = Q(\xi, \eta) \qquad (6)$$

These equations are transformed to (ξ, η) co-ordinates by interchanging the roles of dependent and independent variables which results following elliptic system of equations:

$$Ax_{\xi\xi} - 2Bx_{\xi\eta} + Cx_{\eta\eta} = -J^2(Px_\xi + Qx_\eta) \qquad (7)$$

$$Ay_{\xi\xi} - 2By_{\xi\eta} + Cy_{\eta\eta} = -J^2(Py_\xi + Qy_\eta) \qquad (8)$$

where,

$$A = x_\eta^2 + y_\eta^2, \quad B = x_\xi x_\eta + y_\xi y_\eta \quad C = x_\xi^2 + y_\xi^2$$

and $J = x_\xi y_\eta + x_\eta y_\xi$
is the Jacobian of transformation.

The source terms P and Q in equations (6) and (7) are considered according to Thomas [12].

$$P = \phi(\xi, \eta)(\xi_x^2 + \xi_y^2) \qquad (9)$$

$$Q = \varphi(\xi, \eta)(\eta_x^2 + \eta_y^2) \qquad (10)$$

Now the vorticity – transport equation (1) in the computational plane are rewritten as:

$$\alpha\omega_{\xi\xi} + 2\beta\omega_{\xi\eta} + \gamma\omega_{\eta\eta} + \delta\omega_\xi + \varepsilon\omega_\eta = \text{Re}[u(\omega_\xi\xi_x + \omega_\eta\eta_x) + v(\omega_\xi\xi_y + \omega_\eta\eta_y)] + \text{Re}\,\omega_t \qquad (11)$$

The stream function equation (2) in the computational plane is rewritten as:

$$\alpha\psi_{\xi\xi} + 2\beta\psi_{\xi\eta} + \gamma\psi_{\eta\eta} + \delta\psi_\xi + \varepsilon\psi_\eta = -\omega \qquad (12)$$

The velocities are rewritten as:

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$$u = \xi_y \psi_\xi + \eta_y \psi_\eta, \qquad v = -(\xi_x \psi_\xi + \eta_x \psi_\eta) \quad (13)$$

and the energy equation in the computational plane is rewritten as:

$$\alpha\theta_{\xi\xi} + 2\beta\theta_{\xi\eta} + \gamma\theta_{\eta\eta} + \delta\theta_\xi + \varepsilon\theta_\eta = \mathrm{Re}\,\mathrm{Pr}[u(\theta_\xi\xi_x + \theta_\eta\eta_x) + v(\theta_\xi\xi_y + \theta_\eta\eta_y)] + \mathrm{Re}\,\mathrm{Pr}\,\theta_t \quad (14)$$

where co-efficient are given by

$$\alpha = \xi_x^2 + \xi_y^2 \qquad \beta = \xi_x\eta_x + \xi_y\eta_y \qquad \gamma = \eta_x^2 + \eta_y^2 \qquad \delta = \xi_{xx} + \xi_{yy} \qquad \varepsilon = \eta_{xx} + \eta_{yy}$$

Thompson, Thames, Mastin (TTM) method of generation of automatic boundary fitted co-ordinate generation system is used to construct the grids of the physical flow field. In this study, finite difference implicit method is used to solve the transformed governing equations.

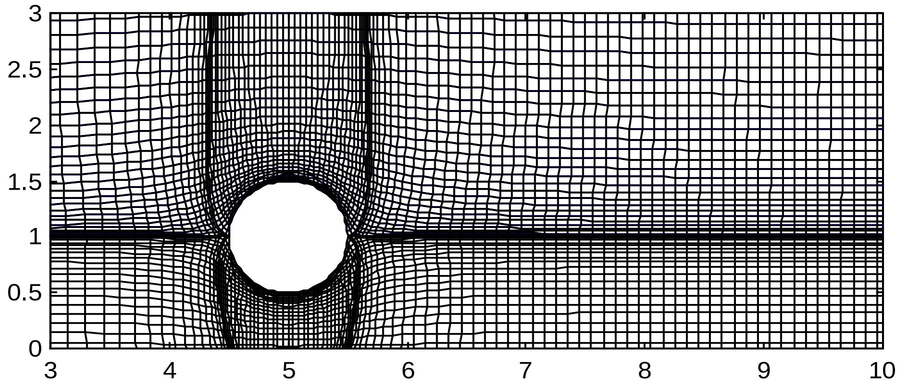

Fig. 2. Mesh of the partial flow domain around cylinder for a gap-ratio of G/D = 0.5.

A typical mesh arrangement (244 x 62) for a gap-ratio of G/D = 0.5 is shown in Fig. 2. Mesh size of (244 x 62) mesh, it is implied that there are 244 nodes in the longitudinal and 62 nodes in the transverse direction, respectively, with 72 nodes on cylinder surface In this study, calculations are performed with a non-dimensional time step of t = 0.002. Care was taken in refining the grid around the cylinder as well as near the wall so that the smaller size of grid is around 0.002 to satisfy the convective stability condition and the diffusive accuracy.

IV. BOUNDARY CONDITIONS

At inlet, uniform longitudinal velocity is considered. No-slip boundary conditions are imposed on upper wall, lower wall and cylinder surfaces. Boundary conditions for vorticity and stream function are described as follows:
The no slip boundary conditions for vorticity at cylinder surface and wall can be written as per Thom's formula, Thom [13].

$$\omega_{i,0} = \frac{2(\psi_{i,1} - \psi_{i,0})}{(y_{i,1} - y_{i,0})^2} \quad \text{at bottom wall} \quad (15)$$

$$\omega_{ic,jc} = \frac{2(\psi_{ic,jc} - \psi_{ic,jc-1})}{h_{ic,jc}^2} \quad \text{at cylinder surface} \quad (16)$$

$$\omega_{i,j} = \frac{2(\psi_{i,j} - \psi_{i,j-1})}{(y_{i,j} - y_{i,j-1})^2} \quad \text{at top wall} \quad (17)$$

where (ic, jc) denotes the nodes on the cylinder surface and $h_{ic,jc}$ is the radial distance between cylinder surface and first body fitted node around the cylinder.

The no penetration boundary condition for u indicates that at cylinder surface:

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$$\psi = \text{constant} \qquad (18)$$

while no-slip boundary condition for u at cylinder surface shows that

$$\frac{\partial \psi}{\partial n} = 0 \quad \text{and} \quad \int_{cyl} \frac{\partial \omega}{\partial n} = 0 \qquad (19)$$

the value of ψ at cylinder is constant , which is updated for unsteady flow at every time step.

V. NUSSELT NUMBER

The local Nusselt number and average Nusselt number around the cylinder are determined by the following expression:

Local Nusselt number : $$Nu = \left(\frac{\partial \theta}{\partial R}\right)_{(R,\bar{\theta})} \qquad (20)$$

The local Nusselt number varies with the angle around the cylinder . The angle $\bar{\theta}$ is measured clockwise from the forward stagnation point as shown in Fig.3.

Mean Nusselt number around the cylinder is determined from the following expression :

Average Nusselt number : $$Nu_m = \frac{1}{2\pi} \int_0^{2\pi} Nu \, d\bar{\theta} \qquad (21)$$

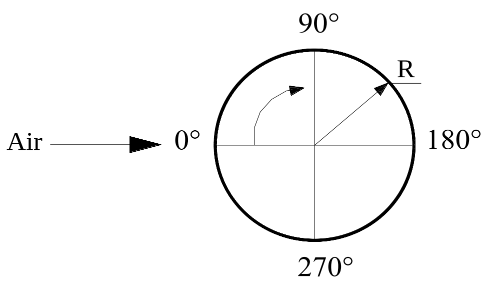

Fig. 3 Position of angle on cylinder surface

VI. RESULTS AND DISCUSSIONS

From the governing equations, it is seen that the heat transfer and fluid flow characteristics depend upon Reynolds number and Prandtl number. In this paper the fluid is considered to be air with a Prandtl number 0.705. The geometrical characteristic is taken into consideration through the introduction of non-dimensional gap-ratio, G/D based on nearest wall from cylinder.

The flow past the cylinder in an unbounded domain to verificate the results on benchmark solutions, can be found from Singha [4]. The effect of the presence of parallel walls on fluid flow and heat transfer, will be considered for different gap ratios.

The flow about a cylinder between parallel walls differs from its unbounded counterpart because of the effect of wall and shear in the incoming velocity profile and separation of vorticity from the nearest wall. The case of a circular cylinder placed between parallel walls at different gaps from nearest wall is studied numerically with the method discussed above.

Vortex shedding : For describing vortex shedding mechanism and interaction with cylinder and walls, a Reynolds number of 100 has been chosen for a gap ratio of G / D =2.0 where G is the gap between the cylinder and the wall and D is the diameter of the cylinder. For G/D = 2.0, the cylinder is located at equal distance from two parallel walls. Time-dependent behavior of streamlines and the mechanism of vortex shedding over a typical time cycle for G/D =2.0 and Re = 100 is shown in Figure 4. The non-uniformity of streamlines around the cylinder strongly suggests the possibility of such an interaction. The flow pattern around the cylinder is essentially periodic in nature i.e. a typical flow pattern is repeated after a fixed interval of time.

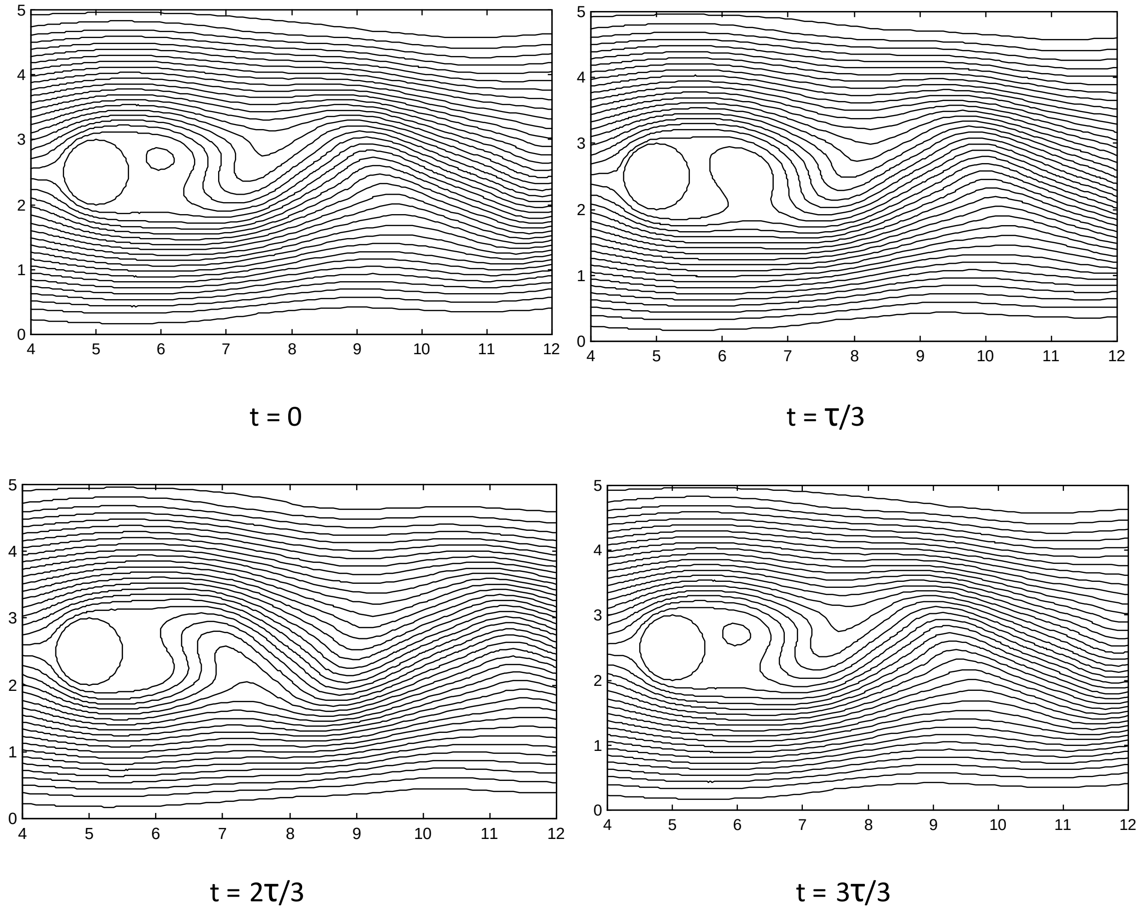

Fig. 4. The vortex shedding across cylinder in a time cycle for Re =100, G/D=2.0.

Fig. 5 shows the time dependent behavior of the stream function at the surface of the cylinder for Re =50, 100 and 150 for G/D=2.0 when the cylinder is at equal distance from both walls. It can be observed that the magnitude of variation grows up as the Reynolds number increases; at the same time, the time period for a cycle decreases, implying that vortices are shed with greater frequency.

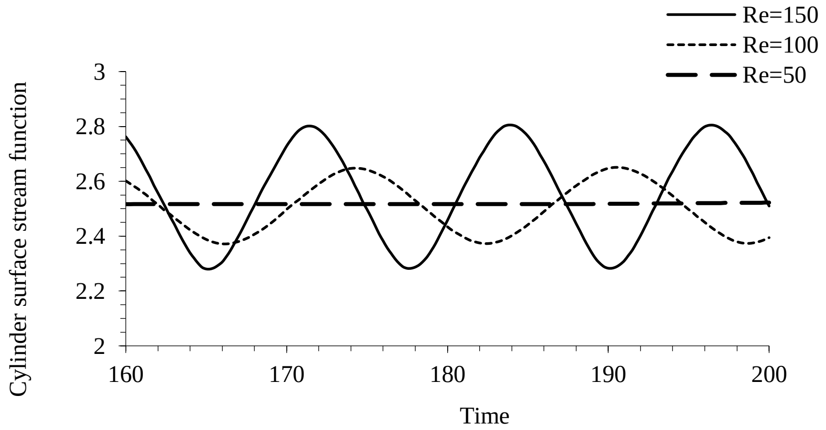

Fig. 5. The time-evolution of cylinder surface stream function value for Re = 50, 100, 150 for G/D=2.0

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The time-evolution of the average Nusselt number at the cylinder surface for different gap ratios and for the Reynolds number Re = 100 is shown in Fig 6. It is seen that when the cylinder is located away from the wall , the average Nusselt number oscillates in a regular manner with the existence of two peaks; a small peak followed by a large peak in one complete time cycle heading for stronger vortex shedding. As the cylinder closed to the nearest wall, the two peak changed to single peak and indicates suppression of vortex shedding. The fluctuation of space averaged, transient Nusselt number is due to the effect of formation-growth-detachment cycle of wake vortices of the cylinder surface.

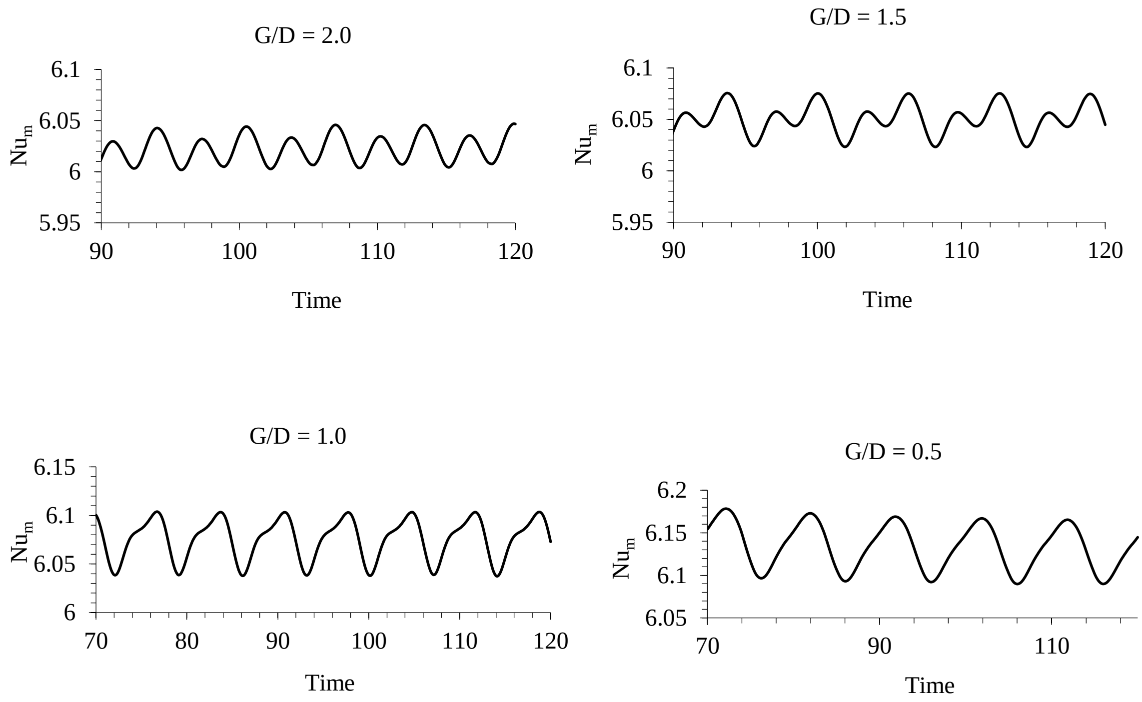

Fig. 6. Time- evolution of average Nusselt number for different gap ratios and Re = 100.

Local Nusselt number : In Fig. 7 variation of local Nusselt number around the cylinder is shown for different gap ratios for Re = 150. It is observed that the local Nusselt number at upper half (say, angles between 0º to about 120º) of the cylinder is nearly constant for all gap ratios. The local Nusselt number at lower half of the cylinder varies as the cylinder comes close to the wall. The local Nusselt number increases in the lower half of the cylinder surface as the gap ratio decreases. The influence of lateral wall seems to increase the velocity of flow and force it to pass through the gap as in case of G/D=0.5. The increase in velocity results in an increase in heat transfer at lower half of the cylinder. The increase in Nusselt number is more prominent at angles between 260º to 320º. In Fig 8, the variation of local Nusselt number for a gap-ratio of 1.0 for different Reynolds number are shown. As expected, local Nusselt number is seen to have higher values with increase in Reynolds number.

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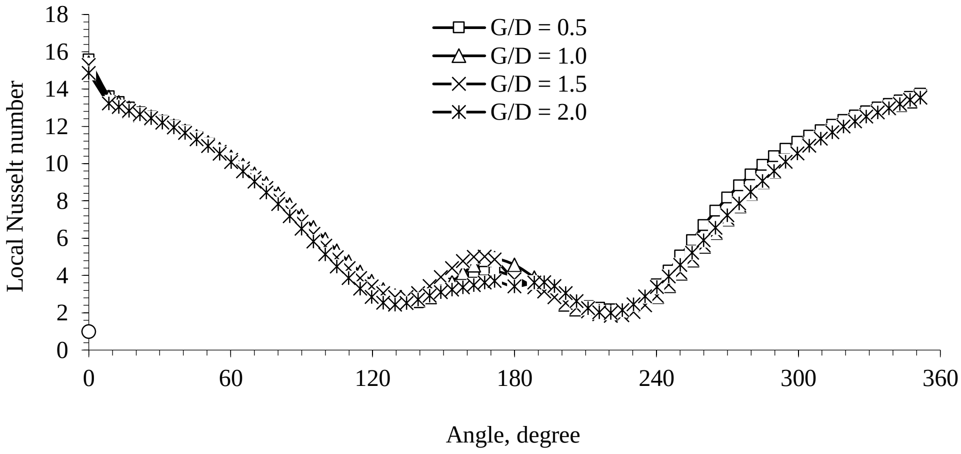

Fig. 7. Local Nusselt number variation for different gap ratios and for Re = 150.

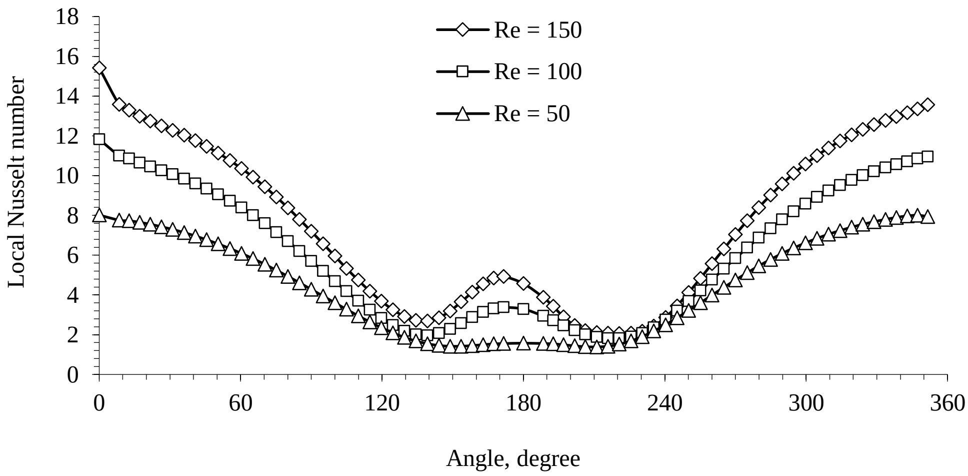

Fig. 8. Variation of local Nusselt number with Reynolds number for a gap-ratio of 1.0.

Average Nusselt number : The variation of time-averaged Nusselt number with the Reynolds numbers are shown in Fig. 9. Reynolds number was set to vary from 50 to 150 while the gap-ratios assumed different values. As expected, the Nusselt number increases monotonically with Reynolds number. Even the presence of wall does not dramatically change the situations with gap-ratio of about 2.0. In this range of gap-ratios, the flow field at the wake of the cylinder is always characterized by shedding of vortices from the top and bottom halves of the cylinder at regular intervals. With lower gap-ratios, the effect of the wall is progressively felt by the cylinder.

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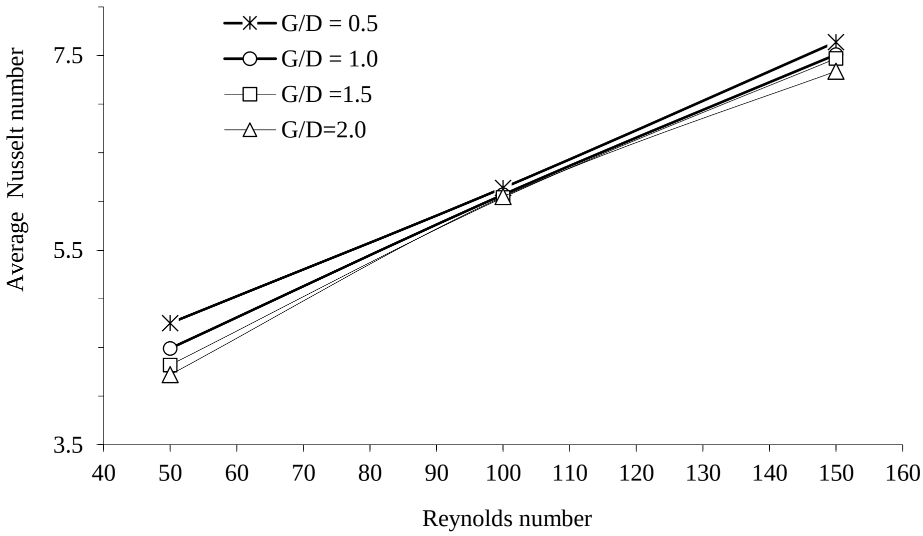

Fig. 9. Variation of average Nusselt number with Reynolds number for different gap- ratios

VII. CONCLUSION

The present study is aimed at the investigation of the fluid flow past a cylinder between parallel walls from a non-isothermal viewpoint. The flow feature was discussed in considerable detail and heat transfer results have been presented. The fluctuation of space averaged, transient Nusselt number of an unbounded cylinder can be explained on the basis of formation-growth-detachment cycle of wake vortices. These vortices have been found affect the cylinder heat transfer. The time-evolution of averaged Nusselt number also provides a measure of frequency of vortex shedding from the cylinder. The proximity of the cylinder to a wall has considerable influence on the flow as well as heat transfer aspects. The proximity of wall seems to increase the velocity of flow and force it to pass through the gap as in case of G/D=0.5. The increase in velocity results in an increase in heat transfer at lower half of the cylinder.

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